Selective Non-Uniform Subdivision

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Abstract

The control meshes of Doo-Sabin and Catmull-Clark subdivision surfaces are usually refined uniformly using a technique called knot-doubling. This uniform refinement approach would perform unnecessary subdivision steps on regions already close to the limit surface enough and, consequently, cause unnecessary (exponential) increase on the number of polygons in the refined meshes. This paper overcomes this problem by proposing a local refinement technique for the control meshes of Doo-Sabin and Catmull-Clark subdivision surfaces. Local refinement is achieved by selectively inserting new knots at midpoints of knot spacings, as for the non-uniform recursive subdivision surfaces (NURSS). Efficient adaptive subdivision can be easily realized based on the new technique.
Key words: *B-splines, Knot insertion, Doo-Sabin surfaces, Catmull-Clark surfaces, NURSS*

1. Introduction

Subdivision surfaces are powerful tools for graphical modeling and animation because of their scalability, numerical stability, simplicity in coding and, especially, their ability to represent complex shapes of arbitrary topology. They have already been used to represent free-form surfaces in several commercial systems. Doo-Sabin and Catmull-Clark subdivision surfaces are two of the most popular subdivision schemes. These subdivision surfaces are based on uniform tensor product B-spline surfaces, whose non-uniform rational extension (NURBS) is an industry standard in computer graphics as well as CAD/CAM systems.

A subdivision surface is the limit surface of a sequence of meshes generated by iteratively refining a given control mesh. The refining process is usually performed uniformly on all the faces of the current mesh using a technique called *knot-doubling*. This uniform subdivision approach would perform unnecessary subdivision steps on regions that are already flat enough and, consequently, cause unnecessary (exponential) increase on the number of faces in the resulting mesh.

In this paper, a technique to solve the above problem is proposed. The technique performs local refinement of a subdivision surface by selectively inserting knots at midpoints of knot spacings. The new technique is named *SNUS* for *selective non-uniform subdivision* because the selective knot insertion process is similar to that of non-uniform recursive subdivision surfaces (NURSS) [10], which generalize non-uniform tensor product B-spline surfaces to arbitrary topology by assigning knot spacings to vertices or edges of the control meshes. Efficient adaptive subdivision can be easily realized based on the new technique as shown in the paper.

The remainder of this paper is organized as follows. Related works in adaptive subdivision for subdivision and parametric surfaces are presented in Section 2. Selective non-uniform subdivision techniques for curves and surfaces are given in Sections 3 and 4, respectively. Concluding remarks and future research directions are given in Section 5.

2 Related work

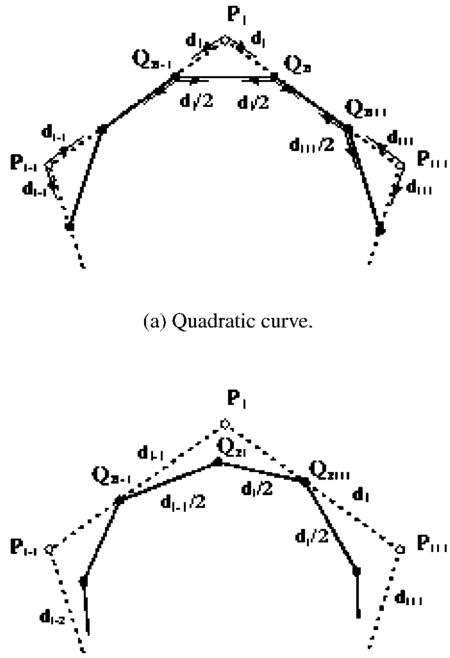
Subdivision defines smooth surfaces as the limit of a sequence of the refinement of polygonal meshes. For regular patches, this sequence can be defined by knot insertion [2, 3, 9]. The Oslo algorithm is well known for the knot insertion scheme for univariate B-splines [3]. Given a set of knots and control points, a function is constructed by control points with breakpoints at the knots. When a new knot is inserted in a knot sequence, the knot insertion scheme updates neighbor control points for representing the same function.

Early subdivision schemes were designed for generalizing this knot insertion scheme to irregular meshes [5, 1, 7]. Since knot intervals were uniformly determined, knots were not represented explicitly. In this meaning, these schemes can be said special cases of knot insertion, in which knots are inserted uniformly at the midpoints of the knot intervals and the number of knots is doubled during each insertion step.

As a subdivision scheme that allows non-uniform knot intervals, Sederberg et al.[10] proposed non-uniform recursive subdivision surfaces (NURSS). They showed that non-uniform knot intervals could be used for controlling the limit surfaces with creases. However, their scheme was still based on knot-doubling, and non-uniform knot intervals were not discussed as a means of adaptive subdivision.

Adaptive subdivision for irregular meshes is one of the future research trends [12]. We believe that knot insertion is one of the most promising schemes for adaptive subdivision.

Another type of approach to adaptive subdivision is to construct schemes that allow for smooth transitions between uniform meshes of different levels. Zorin et al.[13] maintained control points of each subdivision step using hierarchical structures, and realized multiresolution editing of hierarchical meshes . He also described variations of adaptive subdivision using the similar approach [14]. Kobbelt [6] and Velho et al. [11] subdivide only locally specified portions of a uniform mesh to adaptive refine areas of interest. However these approaches do not have the piecewise functional representations that makes analyzing B-splines easier[12].



(a) Quadratic curve.

(b) Cubic curve.

Figure 1. Non-uniform B-spline curve.

3. Curve SNUS

3.1. Curve Knot-Doubling

The control polygon of a periodic B-spline subdivision curve is refined by repetitive *knot-doubling*. *Knot-doubling* here refers to the process of inserting a new knot at the midpoint of each current knot interval[10]. This process doubles the number of control points that represents the same curve. For a non-uniform quadratic periodic B-spline curve, each vertex of the control polygon corresponds to a single quadratic curve segment and a knot interval d_i is assigned to each control vertex $\boldsymbol{P}_i$. A knot-doubling process in this case generates the following new control points $\boldsymbol{Q}_k$:

$$\boldsymbol{Q}_{2i} = \frac{(d_i + 2d_{i+1})\boldsymbol{P}_i + d_i\boldsymbol{P}_{i+1}}{2(d_i + d_{i+1})} \quad (1)$$

$$\boldsymbol{Q}_{2i+1} = \frac{d_{i+1}\boldsymbol{P}_i + (2d_i + d_{i+1})\boldsymbol{P}_{i+1}}{2(d_i + d_{i+1})} \quad (2)$$

as shown in Figure 1(a).

For a periodic cubic B-spline curve, each edge of the control polygon corresponds to a single cubic curve segment and the knot intervals are assigned to its edges instead of its vertices. New control points $\boldsymbol{Q}_k$ in this case are calculated by

$$\boldsymbol{Q}_{2i+1} = \frac{(d_i + 2d_{i+1})\boldsymbol{P}_i + (d_i + 2d_{i-1})\boldsymbol{P}_{i+1}}{2(d_{i-1} + d_i + d_{i+1})} \quad (3)$$

$$\boldsymbol{Q}_{2i} = \frac{d_i\boldsymbol{Q}_{2i-1} + (d_{i-1} + d_i)\boldsymbol{P}_i + d_{i-1}\boldsymbol{Q}_{2i+1}}{2(d_{i-1} + d_i)} \quad (4)$$

as illustrated in Figure 1(b).

The above non-selective subdivision scheme by knot-doubling is problematic where the knot intervals are equal to 0 or much smaller than the other intervals. For example, if one of the knot intervals d_i of a quadratic B-spline curve is equal to 0, Equations (1) and (2) give us $\boldsymbol{Q}_{2i-1} = \boldsymbol{Q}_{2i} = \boldsymbol{P}_i$, i.e., the two consecutive control vertices are identical. This means that the number of control vertices would increase but the curve would not be refined. This is the result of inserting a knot into the joint of two adjacent segments. Further knot-doubling processes there would actually slow down the convergence to its limit curve because it would only accumulate control vertices at the same location. Similar problem would also occur around vertices whose knot intervals are much smaller than the others.

For a cubic curve, the similar accumulation of control vertices is unavoidable on an edge whose knot interval d_i is equal to 0 or much smaller than the others. Equations (3) and (4) are simplified as follows when $d_i = 0$:

$$\boldsymbol{Q}_{2i+1} = \frac{d_{i+1}\boldsymbol{P}_i + d_{i-1}\boldsymbol{P}_{i+1}}{(d_{i-1} + d_{i+1})} \quad (5)$$

$$\boldsymbol{Q}_{2i} = \frac{\boldsymbol{P}_i + \boldsymbol{Q}_{2i+1}}{2} \quad (6)$$

The above equations tell us that $\boldsymbol{Q}_{2i-1}$, $\boldsymbol{Q}_{2i}$, and $\boldsymbol{Q}_{2i+1}$ are on the same edge and the middle point $\boldsymbol{Q}_{2i}$ does not contribute to the refinement.

3.2. Selective Knot Insertion

A simple solution to the above problem is not to insert a knot into the joint of two consecutive segments or into a small knot interval segment selectively. For a quadratic curve, you should not 'cut a corner' to stop inserting a knot and it is straightforward to select effective knot insertions if you have appropriate criteria. The cubic curve case is slightly more complicated, but still straightforward enough as is explained below.

As shown in Figure 2, a knot insertion at the midpoint of the initial knot interval d_i of the non-uniform cubic B-spline curve can be achieved by the following update equations (cf. [9]):

$$\boldsymbol{R}_{i-1} = \frac{d_{i+1}\boldsymbol{P}_{i-1} + \{2(d_{i-1} + d_i) + d_{i+1}\}\boldsymbol{P}_i}{2(d_{i-2} + d_{i-1} + d_i)} \quad (7)$$

$$\boldsymbol{R}_i = \frac{(d_i + 2d_{i+1})\boldsymbol{P}_i + (d_i + 2d_{i-1})\boldsymbol{P}_{i+1}}{2(d_{i-1} + d_i + d_{i+1})} \quad (8)$$

$$\boldsymbol{R}_{i+1} = \frac{\{d_i + 2(d_{i+1} + d_{i+2})\}\boldsymbol{P}_{i+1} + d_i\boldsymbol{P}_{i+2}}{2(d_i + d_{i+1} + d_{i+2})} \quad (9)$$

A midpoint insertion for the next initial knot interval d_{i+1} generates three new control vertices $\boldsymbol{S}_i$, $\boldsymbol{S}_{i+1}$, and $\boldsymbol{S}_{i+2}$. Simple algebra shows that they are given by the same equations as Equations (4), (3), and (9)[1], respectively. Further insertions can be performed by applying the updating process of the control vertices described by these equations.

Note that Equations (7) and (9) are similar to Equations (1) and (2) in the sense that the new vertices $\boldsymbol{R}_{i-1}$ and $\boldsymbol{Q}_{2i}$ are moved from the original vertices $\boldsymbol{P}_i$ and $\boldsymbol{P}_{i+1}$ to new locations on edge $\boldsymbol{P}_i\boldsymbol{P}_{i+1}$. Even though we perform another midpoint insertion for the previous knot interval to the unsubdivided knot interval in the other direction, shown in Figure 2 as green line segments, the location of $\boldsymbol{R}_{i-1}$ remains the same. We will use these facts later in the selective subdivision for Catmull-Clark surfaces.

4. Surface SNUS

Based on the arguments of the selective subdivision of the non-uniform B-spline curves, we will develop a

[1] Equation (9) should be applied for the segment $\boldsymbol{P}_i\boldsymbol{P}_{i+1}$ instead of $\boldsymbol{P}_{i+1}\boldsymbol{P}_{i+2}$.

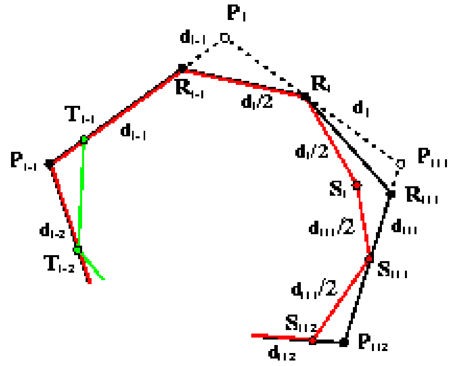


Figure 2. Selective knot insertion.

technique to selectively subdivide Doo-Sabin and Catmull-Clark surfaces to refine their meshes in this section. Even though the original surface is a uniform type, the SNUS performs subdivisions as for the non-uniform recursive subdivision surface.

4.1. Doo-Sabin NURSS

The Doo-Sabin NURSS is an extension of the standard uniform Doo-Sabin subdivision surface and its refinement rules correspond to knot-doubling of bi-quadratic non-uniform tensor product B-spline surfaces. The surface is generated from a polyhedral control mesh by successively cutting off its corners and edges. In the NURSS case, the knot spacings are supposed to be assigned to the vertices since the surface is essentially bi-quadratic, but the control mesh may have vertices whose valence are other than four. Hence the knot spacings are assigned to half edges.

4.2. Selective Subdivision for Doo-Sabin surface

If the control mesh happens to be a rectangular grid, it is straightforward to apply the curve selective subdivision technique to the surface. You should not 'cut a corner' where a knot spacing is not necessary to be inserted. By the selective subdivision, there are four types of the corner cuttings: 1) cutting in the two parameter directions (the same as the original), 2) cutting in one parameter direction, 3) cutting in the other parameter direction, 4) no cutting. In the 2nd case we can compute the new vertices conceptually by applying Equations (1) and (2) twice in one direction after the other.

For a face whose valence is not equal to 4, after one iteration, the new vertices are classified into 4 types as the regular grid case as shown in Figure ??. If the two half edges $\boldsymbol{P}_i\boldsymbol{P}_{i-1}$ and $\boldsymbol{P}_i\boldsymbol{P}_{i+1}$ starting from a vertex $\boldsymbol{P}_i$ of the face should be subdivided, it is regarded as the normal cutting around the vertex and the position of the new vertex is computed by Equation (??). If one of them, say $\boldsymbol{P}_i\boldsymbol{P}_{i+1}$ should not be subdivided, it is treated as the curve selective subdivision and the vertex is calculated by the equations corresponding to Equations (1) and (1).

For simplicity, we assume that all vertices have valence four, which can be achieved through one step of the Doo-Sabin subdivision. As shown in Figure 3(a), the Doo-Sabin subdivision generates four new vertices for each old vertex, one for an adjacent face. We refer to the operation which splits a vertex into four as a vertex-split operation. New faces are created using new vertices as shown in the figure.

In the Doo-Sabin subdivision, the positions of new vertices for regular meshes are determined by (9,3,3,1) masks. Unfortunately, we observed that it was difficult to provide a uniform subdivision mask that allowed selective subdivision using only current vertex positions and knot spacings. Therefore, we simplified the subdivision rule by regarding every face as a parallelogram. As shown in Figure 4.2(b), new vertices are created at the midpoint between the centroid of the face and the old positions. In this case, subdivision masks for regular meshes are described as (5,1,1,1).

We discuss selective subdivision based on this simplified version of subdivision. In our scheme, a knot spacing d_i is assigned to each vertex v_i, and its initial value is assumed to be 1. When vertex v_i is split, knot spacings of newly created vertices are specified to be $d_i/2$.

For selective subdivision, we introduce the following subdivision rules for adjacent faces:

$$\boldsymbol{V} = \frac{\sum_{i=1}^{n} \frac{1}{d_i}\boldsymbol{P}_i}{\sum_{i=1}^{n} \frac{1}{d_i}}, \quad \boldsymbol{P}_i^{new} = \frac{\boldsymbol{V} + \boldsymbol{P}_i}{2}, \quad d_i^{new} = \frac{d_i}{2}$$

These simple rules do not change the positions of centroids of adjacent faces. For example, when vertex A in Figure 4(a) is selectively split and face A^1BCD is created, the positions of Vs for faces $ABCD$ and A^1BCD do not change. Therefore, the position of B^1, C^1, and D^1 can be correctly obtained after A is selectively split.

Figure 4(b) shows new faces created by the vertex-split operation. Numbers shows the values of knot spacings. For selective subdivision, triangle holes appear and they must be filled with new faces. We call such triangle faces as *intermediate faces*, and a corner of each face as an *intermediate corner*, which is shown by a triangle mark in Figure 4(b).

For describing the algorithm for the vertex-split operation, we define a few notations. The operation that splits v is described as $split(v)$. A set of vertices that are connected to v are referred to as $star(v)$. If vertex v_i contains intermediate corners, $marked(v_i)$ becomes true. Then, the algorithm for splitting vertex P in Figure 5 can be described as follows:

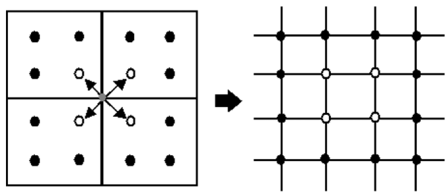
1. If vertex v_i is included in $star(P)$, and $marked(v_i)$ is true, v_i is split before P. $split$ operations can be called recursively.

2. Four new vertex positions are calculated using four adjacent faces that are not *intermediate*. In Figure 5(b), vertices P_1, P_2, P_3, and P_4 are created.

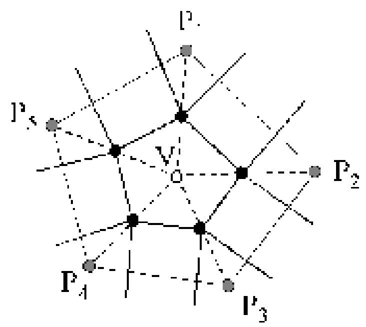
3. Faces around P are referred to as $f_1, f_2, .., f_5$. If f_i is not *intermediate*, the vertex P of f_i is replaced by the new vertex P_i. Otherwise, vertex P of f_i is replaced by the two new vertices calculated for two adjacent faces. In Figure 5(c), the positions of Ps of f_1, f_2, f_3, and f_4 are modified, and f_5 is converted into a quadrilateral face.

4. Triangle faces are created to fill holes. Marks are added at the corners of new triangle faces, as shown in Figure 5(d).

Figure 6 shows an example in which 18 faces are subdivided into 451 faces using our selective subdivision scheme. In this subdivision, vertices are split in order of the distance between the current position and the limit position.



(a) New vertices.



(b) Simplified subdiv.

Figure 3. Doo-Sabin subdivision.

Figure 8 shows an advantage of our method. In this model, the uniform Doo-Sabin subdivision exponentially increases the original 413 faces to 1480, 5898, 23562 faces, and so on. In our method, the model can be subdivided in

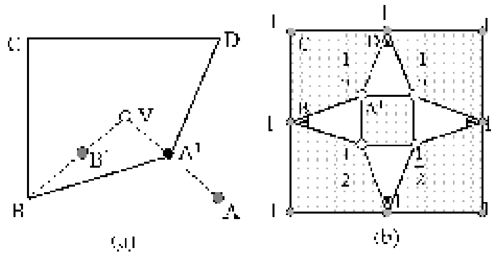


Figure 4. **New vertices and faces: (a) New positions of A and B. (b) Intermediate faces and corners.**

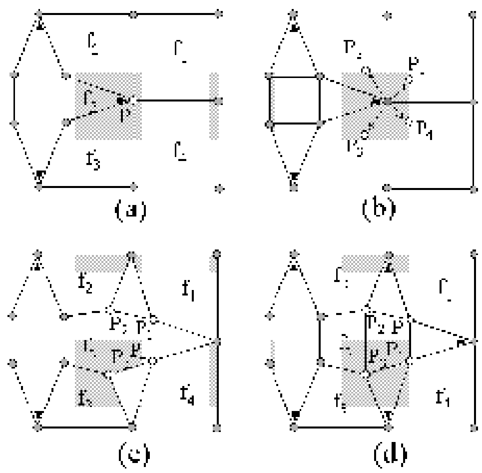


Figure 5. **A sequence for realizing selective subdivision.**

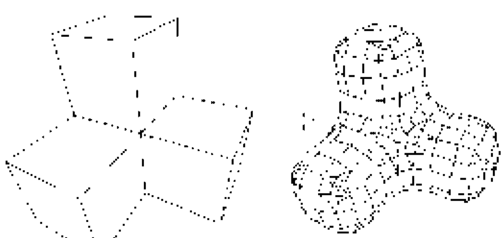


Figure 6. **An example of selective subdivision. (left:18 faces, right:541 faces).**

any number of faces, as shown in this figure. Therefore, users can easily select enough quality for their purposes.

4.3. Catmull-Clark NURSS

As the Doo-Sabin NURSS, the Catmull-Clark NURSS extends the standard uniform Catmull-Clark subdivision surface and its refinement rules correspond to knot-doubling of bi-cubic non-uniform tensor product B-spline surfaces. The surface is refined by splitting faces and averaging vertices. The knot spacings are assigned to the edges for the Catmull-Clark NURSS since the surface is bi-cubic in essence.

4.4. Selective Subdivision for Catmull-Clark surface

Our implementation of the SNUS for the Catmull-Clark surface has the following properties: 1) For a regular mesh, the faces can be subdivided into two or four faces along their isoparametric lines. 2) The faces which have extraordinary points are always subdivided into four faces together with all other faces around their extraordinary points if necessary. 3) The limit points of the all vertices are guaranteed to be on the limit surface of the original mesh.

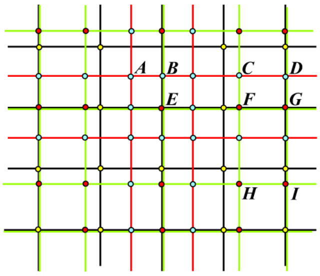
4.4.1 Knot insertion to B-spline surface

Before explaining the SNUS for the Catmull-Clark surface, at first we will analyze the relocation types of the vertices of the control mesh of a non-uniform Bi-cubic B-spline surface by inserting knots at midpoints of knot spacings to understand our SNUS algorithm.

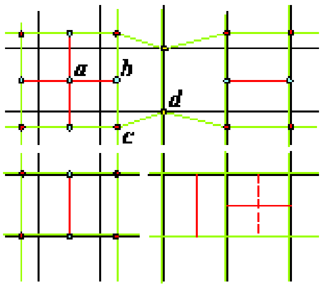
By inserting two knots for each parameter directions, the control points of a B-spline surface will become as shown in Figure 7(a). The original mesh in black color are changed to the new mesh in green by inserting knots along the isoparametric lines in red. The vertices in yellow of the original mesh are moved to new positions indicated by red points. The blue points shows the positions of newly created vertices. There are 9 types of their positions determined by the combinations of Equations (3), (4), and (7)(or (9)). For example, the position of vertex $\boldsymbol{E}$ (the vertex point in the case of the NURSS Catmull-Clark subdivision) can be determined by applying Equation (4) in the both parameter directions. In other words, if we insert knots selectively, the face point is only of one type ($\boldsymbol{A}$), the edge point 3 types ($\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$), and the vertex point 5 types ($\boldsymbol{E}$-$\boldsymbol{I}$).

4.4.2 Algorithm

We assume that all faces are quadrilateral, which can be achieved through one step of the Catmull-Clark subdivision. As shown in Figure 7(b), the subdivision patterns of



(a) Knot insertion to B-spline surface.



(b) Subdivision patterns.

Figure 7. Subdivision patterns.

the faces are classified into 1) a face is subdivided into four subfaces: *four-face* subdivision, and 2) a face is subdivided into two subfaces: *two-face* subdivision. Their vertices are calculated or updated assuming that a knot is inserted at the midpoint of the knot spacing assigned to the edges of the face to be subdivided. The vertices of the subdivided faces can be classified into one of vertices from $\boldsymbol{A}$ to $\boldsymbol{I}$ and their locations are obtained.

As shown in the right-bottom corner of Figure 7(b), if a two-face subdivision is supposed to be adjacent to another two-face subdivision in the other parameter direction or a four-face subdivision, the locations of the vertices can not be determined in such cases. Hence such subdivision patterns are not allowed and if such a subdivision is required, both of the faces are subdivided by four-face subdivision.

The types of the vertices are classified into 1) normal, 2) t-shape, 3) shallow, and 4) deep. The t-shape is a vertex generated between subdivided and unsubdivided faces and whose valence is equal to 3 ($\boldsymbol{b}$ in Figure 7(b)). The shallow(deep) is a vertex whose subdivision depth is shallower(deeper) than one of the vertices connected to it. Vertex $\boldsymbol{d}$ is classified as "swallow" because it is shallower that vertex $\boldsymbol{c}$. Hence vertex $\boldsymbol{c}$ is deep. The other vertices are regarded as normal. The vertices of the initial mesh are all normal vertices. Vertex $\boldsymbol{a}$ is also normal because the vertices connected to it are generated or relocated by the same number of subdivisions. The distinction between the normal and the others is important because if the mesh includes t-shape, shallow or deep vertices, we do not perform subdivision around them. We could perform a NURSS subdivision, but the limit surface would become different from that of the original mesh.

The SNUS algorithm for the Catmull-Clark surface is summarized as follows:

1. As a preparatory step, 1 is assigned to all edges of the initial mesh.

2. Select edges to be subdivided by some criteria, such curvature around the vertices connected to them.

3. The edges selected in the previous step are paired with two opposite sides and attach a subdivision flag to all these edges.

4. A face which has at least one selected edge is subdivided with two-face or four-face subdivision according to the total number of edges with the subdivision flag and t-shape vertices included in the face. If the number is equal to two or four, a two-face or four-face subdivision is performed. Before subdividing faces, the new locations of face, edge, and vertex points and their limit points are calculated and attached to the faces, edges, and vertices, respectively. For the edges and vertices, all possible locations are attached for further subdivisions.

5. In case that the vertices of a selected edge or at least one of its paired edges is t-shape, at first t-shape vertices are converted to normal by subdividing the adjacent face with two-face or four-face subdivision according to the state of the face.

6. Repeat steps from 2) to 5) if necessary.

Note that the above algorithm does not essentially increase the number of extraordinary points through the subdivision process. The t-shape vertices are generated during subdivision, but they are supposed to be converted to normal vertices by further subdivisions.

Figure 9 shows two illustrative examples applied to uniform Catmull-Clark surfaces. (a) and (b) are meshes in the subdivision process generated from the same initial mesh and the vertices in (b) and (d) are moved to their limit points. They are located on the limit surface of the initial mesh. Because of the difference of the criteria applied to them, they are differently subdivided. Figure 10 shows the same monster head in Figure 8, but it is used as a Catmull-Clark surface. The criterion we used here is angles between each edge and the normals at its vertices. One and two SNUS subdivision steps produce meshes (b) and (c). (d) is a shaded image of the mesh (c). The numbers of the faces of (b) and (c) are 4301 and 7037 and those of the corresponding fully subdivided meshes are 4712 and 18848, respectively.

4.5. SNUS in parameter space

We can apply our technique naturally to NURSS surfaces in the parameter space and use the values of the knot spacings as a criterion for selecting knot insertion locations. One of the typical examples which definitely need SNUS is degree-elevated subdivision surfaces. The converted surface is represented with multiples knots and many of its knot spacings are equal to 0. The SNUS can avoid inserting knots to the mesh where the knot spacings are equal to 0 or relatively very small.

Figure 11 shows Catmull-Clark surface examples subdivided with the SNUS in the parameter space. In the figure, red and yellow edges are derived from those of the converted mesh from Doo-Sabin to non-uniform Catmull-Clark surfaces[8]. The initial knot spacings of the red and yellow edges are supposed to be 0 and 1, respectively. To clarify the effect of the SNUS, 0.1 is assigned to the red edges instead of 0. The standard subdivision generate meshes shown in Figure 11(a) and (c) and a lot of polygons are accumulated around the red edges without much improvement of the shape according to the subdivision depth. If the original value 0 is assigned to the red edges, polygons degenerated to a point or a line are produced. The SNUS can avoid such accumulations of unnecessary polygons. The sizes of the files (obj) of those meshes are (a) 1.66, (b) 0.64, (c) 6.82, and (d) 2.11 MBytes. The sizes of the meshes by the SNUS are about one third of those generated by the standard subdivision.

5 Conclusion

This paper presents a new technique, the SNUS, for local refinement of Doo-Sabin and Catmull-Clark subdivision surfaces, which selectively inserts new knots at midpoints of knot spacings, as for the non-uniform recursive subdivision surfaces (NURSS).

To explore the capabilities of the SNUS, we have developed several types of SNUS based adaptive subdivision methods for Doo-Sabin and Catmull-Clark surfaces. In the Doo-Sabin subdivision, we proposed a vertex-split operation that enabled selective subdivision only using current vertex positions and knot spacings. Our system allows to subdivide models into the specified number of faces instead of the exponential increase. In the Catmull-Clark subdivision, our method subdivides a face into two or four subfaces along their isoparametric lines. It does not increase the number of extraordinary points. In the both selective subdivision schemes, the limit points of the all vertices are guaranteed to be on the limit surface of the original mesh.

One of the future research topics is on local refinement by inserting knots at arbitrary positions instead of midpoints. Since the quality of adaptive subdivision depends heavily on subdivision criteria. Additional work should be devoted to such criteria to extract the maximum power of the SNUS.

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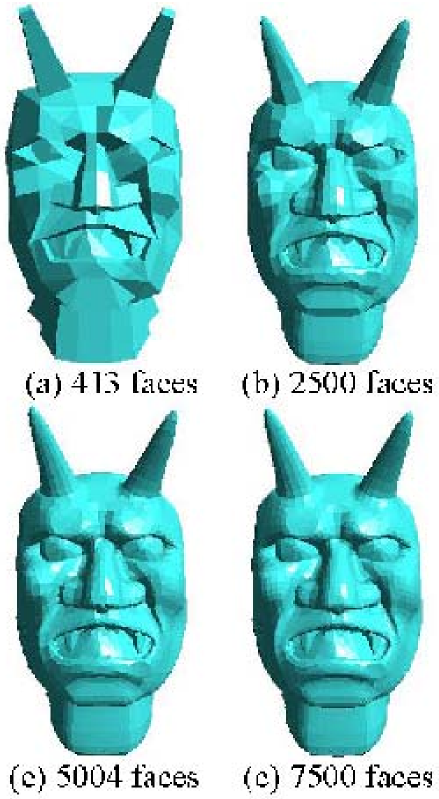
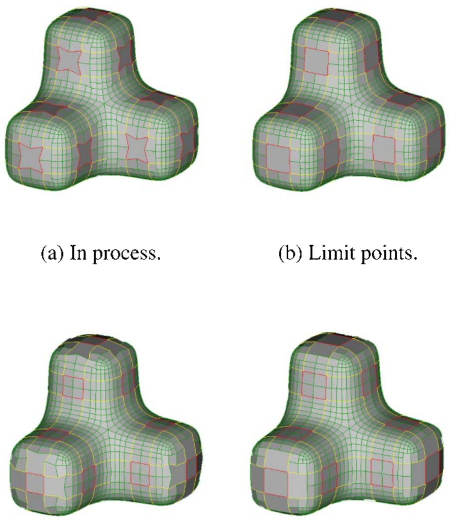


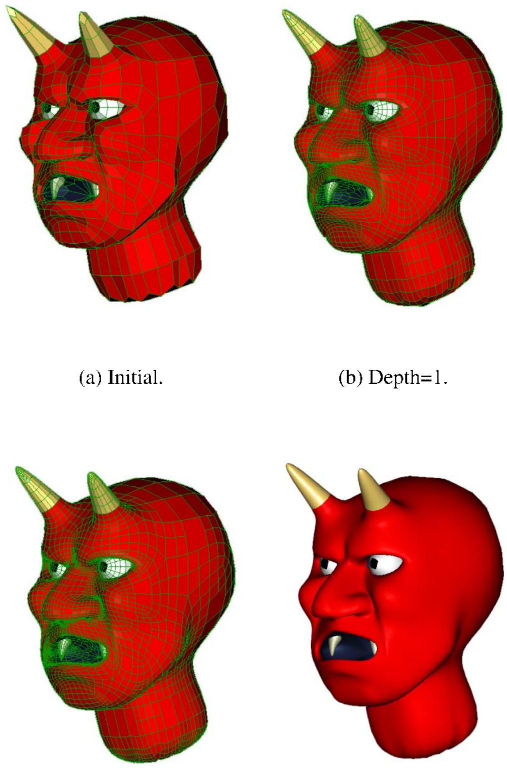
Figure 8. **Selective subdivision into specified numbers of faces.**



(a) In process. (b) Limit points.

(c) In process. (d) Limit points.

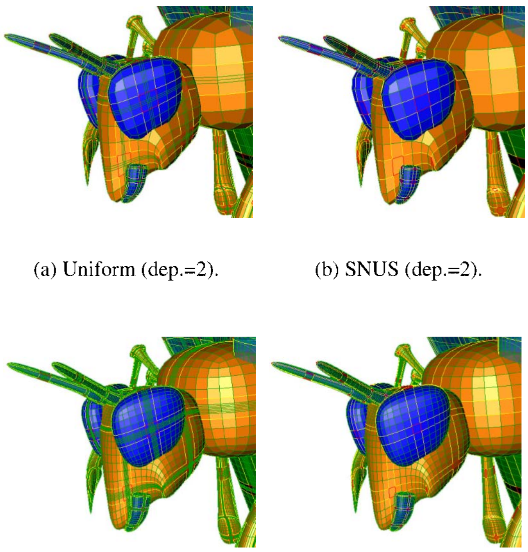
Figure 9. **SNUS for a uniform Catmull-Clark surface.**



(a) Initial. (b) Depth=1.

(c) Depth=2. (d) Shading.

Figure 10. **SNUS for a uniform Catmull-Clark surface.**



(a) Uniform (dep.=2). (b) SNUS (dep.=2).

(c) Uniform (dep.=3). (d) SNUS (dep.=3).

Figure 11. **SNUS in parameter space.**