Interactive Deformation Using Volumetric Constraints

Kentaro Doba, The University of Tokyo, doba@nakl.t.u-tokyo.ac.jp

Kenta Ogawa, The University of Tokyo, kogawa@nakl.t.u-tokyo.ac.jp

Kenji Murakami, The University of Tokyo, muranken@nakl.t.u-tokyo.ac.jp

Hiroshi Masuda, The University of Tokyo, musuda@nakl.t.u-tokyo.ac.jp

1 Introduction

We present novel mesh deformation techniques using volumetric constraints. Firstly, we propose a fast volume-preserving deformation method for 2D closed meshes. Then, we introduce constrains that preserve the internal structure of 3D meshes.

2 Deformation using Volumetric Constraints

2.1 Fast volume-preserving deformation

Since volume-preserving deformation requires solving non-linear equations, special care must be taken for interactive deformation. [Huang et al. 2006] enclosed a mesh model with a single sparse lattice and reduced the number of variables by representing all vertex positions by linear combinations of points in the lattice. Then, constraints can be represented using small dense matrices. In their method, the lattice must be sparse because the cost of the decomposition of a dense matrix is $O(N^3)$. This limitation restricts the degrees of freedom for mesh deformation.

In our method, we enclose a mesh using multiple overlapping lattices so that some vertices are shared by two or more lattices, as shown in Figure 1. Such shared vertices mediate to propagate deformation between disconnected lattices. Suppose the coordinates of the lattices A and B are $\{\mathbf{p}_{i(A)}\}$ and $\{\mathbf{p}_{i(B)}\}$. When vertex $\mathbf{X}_{i(A,B)}$ is enclosed by the two lattices, the vertex position can be represented in two ways using mean value coordinates [Ju et al. 2005]:

$$\mathbf{x}_{i(A,B)} = \sum_j w_{ij(A)}\mathbf{p}_{j(A)} = \sum_k w_{ij(B)}\mathbf{p}_{k(A)} \qquad (1)$$

To smoothly propagate deformation between the lattices, we represent $\mathbf{X}_{i(A,B)}$ as:

$$\mathbf{x}_{i(A,B)} = \alpha\sum_j w_{ij(A)}\mathbf{p}_{j(A)} + (1-\alpha)\sum_k w_{ik(B)}\mathbf{p}_{k(B)} \qquad (2)$$

where $\alpha = d_B / (d_A + d_B)$; d_A and d_B are the distances from $\mathbf{X}_{i(A,B)}$ to each lattice.

Volume-preserving deformation can be calculated by solving the following constrained minimization:

$$\min \|\mathbf{AWp} - \mathbf{b}\|^2 \text{ subject to } g(\mathbf{Wp}) = 0 \qquad (3)$$

where $\mathbf{p}$ is the vertex positions of lattices, $\mathbf{Ax}=\mathbf{b}$ represents linear Laplacian equations for the mesh, $\mathbf{W}$ represents mean value coordinates $\{w_{ij}\}$, and $g(\mathbf{Wp})=0$ is a constraint for preserving the volume. In our method, $\mathbf{W}$ has the structure shown in Figure 2. Since this matrix is relatively sparse, Eq.(3) can be quickly solved using Cholesky decomposition even when the total number of points in lattices becomes large to increase the degrees of freedom for deformation.

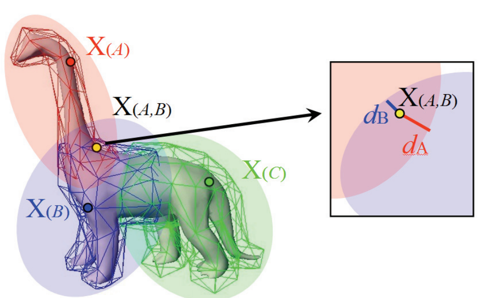


Figure 1: Mesh model enclosed by three lattices

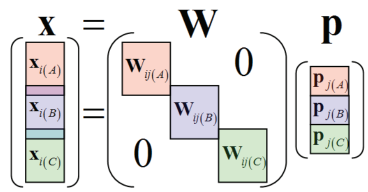


Figure 2: Structure of **W**

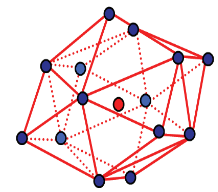


Figure 3: Interior vertex

2.2 Deformation of 3D mesh models

A 3D mesh, which consists of tetrahedra, has internal structure, in which attributes may be embedded. To deform internal structure consistently, we introduce simple linear constraints. Since each interior vertex is enclosed by adjacent tetrahedra, as shown in Figure 3, its position can be represented by a weighted sum of the positions of the connected vertices using mean value coordinates:

$$\mathbf{p}_i - \sum_{j \in N(i)} w_{ij}\mathbf{p}_j = 0 \qquad (4)$$

By adding these constraints in matrix $\mathbf{A}$ in Eq.(3), the internal structure can be also maintained during deformation.

3. Result

Figure 4 shows the computation time of our volume-preserving deformation in the interactive phase. We solved non-linear minimization using the Gauss-Newton method. In this experiment, we used the same total number of points for lattices, but subdivided the enclosed space into the different number of overlapping regions. This result indicates that the computation becomes significantly fast when the number of lattices increases.

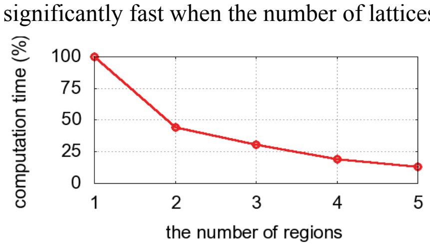
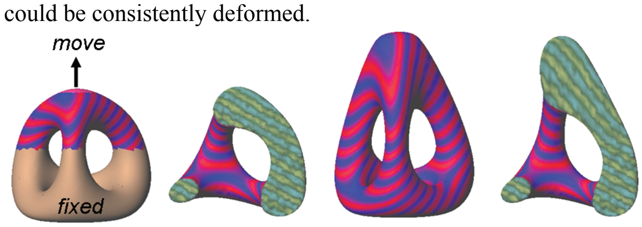


Figure 4: Computation time

Figure 5 shows an example of the 3D mesh deformation. Cross-sections are shown to visualize the interior. As shown in this figure, both the boundary and interior of the 3D mesh model could be consistently deformed.



(a) Original mesh and section (b) Deformed mesh and section

Figure 6: Deformed result of 3D mesh

References

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